

The Probability of Paternity

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INTRODUCTION

Much has been written on how to calculate the probability of paternity when testing fails to exclude the alleged father from being the true father of a child, and it is clear from what has been written that the concept of "probability" is often not well understood. I have therefore decided to focus my talk specifically on this problem. I shall review the principles and assumptions underlying the "calculation" of the probability of paternity, which are the same whether DNA or non-DNA markers are used, and give you my opinion of how, without scientific fraud, this information should be transmitted in court.

PROBABILITY

There are many ways of defining probability, but the frequency definition, i.e., defining probability to be a limiting frequency ratio on repeated sampling from the same population, is the only definition that we can be sure has any relevance to the real world. I shall take as axiomatic what Fisher (1959) wrote on this topic:

"It has been often recognized that any probability statement, being a rigorous statement involving uncertainty, has less factual content than a assertion of certain fact would have, and at the same time has more factual content than a statement of complete ignorance. The *knowledge* required for such a statement refers to a well defined aggregate, or population of possibilities within which the limiting frequency ratio must be exactly known. The necessary *ignorance* is specified by our inability to discriminate any of the different subaggregates having different limiting frequency ratios, such as must always exist ...

"... although in the deduction of statements of certainty it is legitimate to draw inferences from some of the axioms available while ignoring others, or, in other words, to base a valid argument on a chosen subset only of the available axioms, no such liberty can be taken with statements of uncertainty, where it is essential to take the whole of the data into account, though some part of it may be shown on examination to be irrelevant, and not to affect the result."

I take this to be axiomatic, not because the majority of those calculating probabilities of paternity would agree with Fisher, but because I have thought long and hard about the problem and have come to the conclusion that Fisher best expounds my point of view in this regard. When we talk about the probability of paternity we are not talking about a future event that will or will not take place as a result of a random process, but rather the probability of an event that has already occurred or not occurred, so that its probability must depend crucially on the amount of information we have that is relevant to that event. For any of the information to be "irrelevant" we must be able to show that it does not affect the resulting probability.

BAYES' THEOREM

The probability of paternity is calculated using "Bayes' theorem" often without regard to the reason why Bayes himself, knowing that it contained a flaw, did not publish his essay on probability. The important thing to remember about Bayes' theorem is that it only allows one to change probabilities, not to create them. Let us first consider an example that has nothing to do with paternity. Suppose a colleague telephones me to say that he is speaking with a man who is either black or white, and he wants me to determine the probability that the man is white. Without further information, there is no way that I can determine such a probability. If, however, he tells me that the man is American, I can immediately use the fact that about 85% of such American men are white and quote him the probability of .85. If, on the other hand, he tells me that the man is a resident of New Orleans, I can use the fact that 45% of the males in New Orleans are white and quote a probability of .45. These probabilities are, in fact, limiting frequency ratios in the sense that if in each case men were repeatedly sampled at random from either America or New Orleans, the frequency ratio of whites would tend towards .85 or .45, respectively.

Suppose my colleague then tells me that the man he is speaking with, in addition to being a resident of New Orleans, attends college. Using the principles enunciated by Fisher, I must use this information if it is relevant. Now, it is well established that in New Orleans at the present time, whereas about 35% of white males go to college, only about 12% of black males do so. Thus the probability that a New Orleanian male is white and attends college is $(.45)(.35)$, while the probability that he is black and attends college is $(.55)(.12)$. Given that one of these two events actually occurred, we must divide each by their sum to arrive at the new probabilities that he is white or black. Thus the probability that the man is black now becomes

$$\frac{(.45)(.35)}{(.45)(.35) + (.55)(.12)} = .7 \quad (1)$$

Note carefully that this is a limiting frequency ratio and has nothing to do with whether the color of one's skin *per se* causes

one to go to college. Note also that we started with a "prior" probability of .45 that the man is white (from our knowledge that he resides in New Orleans) and then, on the basis of the further observation that he is attending college, we modified this prior probability to arrive at a "posterior" probability of .7.

The probability of paternity is calculated in an exactly analogous manner. Whereas in the above example, we started with the knowledge that either the man is white or he is black, in this case we start with the knowledge that either the alleged father is the father of the child or somebody else is the father. If we know the prior probabilities of these two possibilities, they can be converted, using the blood testing results, into what is known as the probability of paternity. Can we assume that if we know absolutely nothing about the prior probabilities, they are equal? This is the flaw that is in Bayes' original essay - the assumption that lack of knowledge somehow implies that the prior probabilities are equal - which Bayes knew was an illogical assumption. Morris (1982) has stated that the assumption of equal prior probabilities is "neutral" - whatever that means. When a die is thrown, it will certainly turn up either "six" or "not six"; is it logical, or even "neutral," to assume that these two events have equal prior probabilities?

It should be fairly obvious that the additional information observed when blood testing is performed comprises the phenotypes (i.e., the results of antigen-antibody reactions or electrophoretic bands) of the mother (M), child (C) and alleged father (A). Let us suppose that the probability of obtaining the particular results found is .35 if the alleged father is the father, but only .12 if some other man is the father. We can then use Bayes' theorem, just as before, and arrive at a probability of paternity of 0.7. Thus, to arrive at a correct probability of paternity, we have to start with a correct prior probability of paternity and we have to modify this with correct probabilities of what we observed under the two possible "hypotheses" - the "alleged father is the father" and "some other man is the father". At this point I want to discuss these last two probabilities in more detail, leaving the prior probability until later.

THE PATERNITY INDEX

It is clear that I can equivalently write the above formulation (1) as follows, dividing through the top and bottom of the fraction by .12:

$$\frac{(.45) \left(\frac{.35}{.12} \right)}{(.45) \left(\frac{.35}{.12} \right) + .55} = .7 \quad (2)$$

This shows that all the information contained in the test data that is relevant to the probability of paternity is given by the fraction .35/.12, known in this case as the paternity index, i.e.,

$$\frac{P(M,C,A \mid \text{alleged father is the father})}{P(M,C,A \mid \text{some other man is the father})} . \quad (3)$$

The paternity index is often equated to

$$\frac{P(M,C,A \mid \text{alleged father is the father})}{P(M,C,A \mid \text{"a random man" is the father})} = \frac{P(C|M,A)}{P(C|M)} , \quad (4)$$

where this last equality assumes random mating. Changing "some other man" to "a random man" is legitimate so long as we understand that by "a random man" we mean a man randomly chosen from a population of all men who could possibly be the father, each weighted by the probability that he is in fact the father (these probabilities being based on information that is known independently of the test data). In other words we have to assume a probability distribution for the phenotypes of all possible fathers (this may or may not be the same as the distribution of phenotypes in a natural living population).

Provided we define the paternity index such that "the random man" appropriately reflects the distribution of phenotypes among all such men, it is appropriate. To my knowledge, the difficulty of ensuring this is rarely stressed by "expert witnesses" who quote probabilities of paternity. Nor do they estimate either the numerator or the denominator of the paternity index as an observed limiting frequency ratio in an actual population. Rather they make assumptions, which again they rarely explain to the courts. The two major assumptions usually made are:

- (1) the observed phenotypes are caused by a set of Mendelian systems whose population distributions are mutually independent
- (2) all allele frequencies of the systems tested are known without error for the population from which the father comes.

(They also assume, for systems in which there is dominance, that the genotypic frequencies in the populations from which the father and the mother come are in Hardy-Weinberg proportions). I wonder what is the nature of the evidence that these are reasonable assumptions in the case of many DNA markers. Cellmark Diagnostics has stated in its literature "The odds that two unrelated people possess the same DNA FINGERPRINT have been calculated to be on average, 30 billion to 1. Given that the Earth's population is about 5 billion people (only 2.5 billion males), it is impossible to be more sure of a paternity determination than this with any other available test." It is obvious that the ratio of 30 billion to 1 could not have been observed in any human population and must have been "calculated" on the basis of certain assumptions. The courts should surely be made aware of all assumptions used in making any such

"calculations," and to what extent their validity has been verified.

RED HERRINGS

Li and Chakravarti (1985, 1986, 1988) have given three reasons why they believe the paternity index, used as I have indicated, is not valid. First, they point out that it is higher among fathers than non-fathers 'for algebraic reasons.' In other words, it is not paternity *per se* that causes the index to be high. This is analogous to saying that it is not the color of one's skin *per se* that causes one to go to college. As we have seen, this is irrelevant. Second, they say that the paternity index "is not a comparison of two non-overlapping alternate hypotheses, as claimed." But the hypotheses "the alleged man is the father" and "some other man is the father" are clearly two distinct, non-overlapping alternate hypotheses. Third, they point out that when using the paternity index it is possible in certain situations to conclude that "the alleged father non-excluded by three tests is *more* likely to be the true father than if he is non-excluded by two *additional* tests." Because they find this possibility unreasonable, Li and Chakravarti recommend that the paternity index be calculated on the basis of observing merely that there has not been an exclusion (Baur et al. 1986; Elston 1986). If we know the phenotypes of the mother, child and alleged father, we certainly know whether or not there has been an exclusion, whereas the converse is not true. Therefore, Li and Chakravarti are not making use of all the information available, which violates the principles so clearly stated by Fisher. I have shown elsewhere (Elston 1986) in what sense their method of calculating the probability of paternity has "validity", but less "utility." Using the principles enunciated by Fisher, however, their probability is not even valid.

THE PRIOR PROBABILITY

Finally, I want to discuss what should be done about the prior probability, without which no probability of paternity can be calculated. At this point a clear distinction should be made between the scientific approach, which is to make a rational inference on the basis of *all* the information available, and the legal approach, which considers certain kinds of information irrelevant - even though it may not be rational to do so. The legal system (in America, at least) operates on the premise that an accused person is innocent until proved otherwise, and that the mere act of being accused should not be construed as suggesting guilt. But the fact that a man has had his blood tested for paternity singles him out from the rest of the population as being someone who comes from the class ("sub-aggregate" in Fisher's language) of all men who are, or have been, tested for paternity. Provided this class is stable over time, a scientist should use past information about this class (which should probably be laboratory-specific) in order to time, a scientist should use past information about this class (which should probably be laboratory-specific) in order to

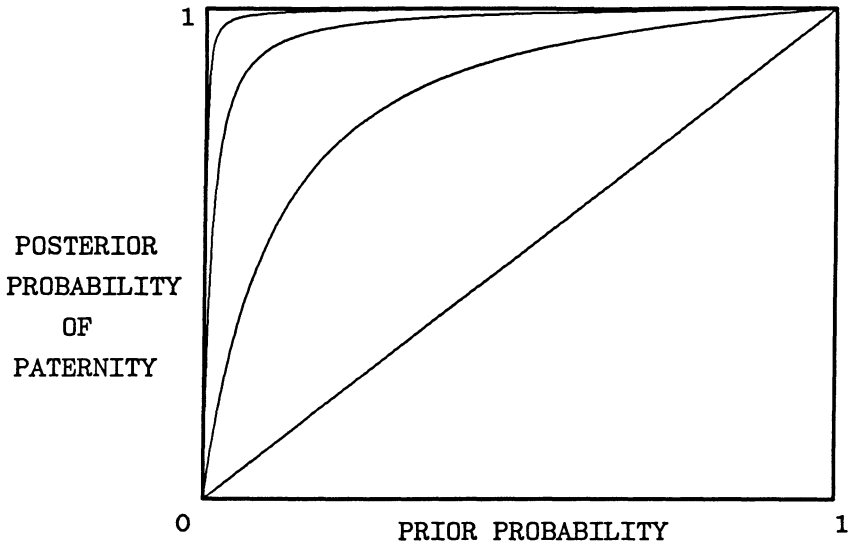


Fig. 1. Plot of the posterior probability of paternity (ordinate) against the prior probability (abscissa) for four values of the paternity index: 1, 10, 100 and 1000, giving rise to a diagonal line and three successively higher lines, respectively.

calculate a reasonable prior probability of paternity. This has been suggested by several authors (see especially Baur et al. 1981). Using such knowledge is analogous to using our knowledge of the physical structure of a die to assume that the prior probability of a "six" is $1/6$. The law, however, assumes that this information is not relevant and so a prior probability based on accusation alone cannot be used. It is therefore up to the courts to decide, on the basis of the *independent* evidence presented, what the appropriate prior probability is. The expert witness who does the blood testing must somehow summarize the evidence from the testing data alone. This could be done by quoting the paternity index (together with all the assumptions made to calculate it) or, and I personally believe this is preferable, by quoting the probability of paternity (again, together with the assumptions made) as a function of the prior probability. This can very easily be done by plotting the probability of paternity against the prior probability over its whole range from 0 to 1. Figure 1 illustrates, for four different values of the paternity index, what this function looks like. In particular, I think it is essential to point out in court that this curve goes through the point (0, 0) whatever the paternity index is. Otherwise a judge who is convinced that the alleged father had no access to the mother may be confused by a high probability of paternity.

CONCLUSION

Anyone who uses scientific expertise to quote a probability of paternity in court has the duty to "tell the whole truth and nothing but the truth." This clearly includes all the assumptions made in calculating the probability, and an evaluation of the validity of these assumptions. Anything less than such a full interpretation is surely both scientific fraud and fraud on the court.

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