

ADVANTAGES OF THE EXCLUSION PROBABILITY IN THE BIostatISTICS  
OF BLOOD GROUP OPINIONS ?

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By exclusion one does not recognize all non-fathers. Hence,  
it is necessary to establish a plausibility for or against  
paternity. If one wishes to use the fact of non-excludability  
as information, one has to work with the term  $1-A$  = non-ex-  
clusion chance for non-fathers.

It represents the frequency for the counter-hypothesis  $Y$   
(= non-paternity)

$$f(Y) = 1-A.$$

The frequency of the null-hypothesis  $X$  (= paternity) is

$$f(X) = 1 \text{ (which means: 100\% non-exclusion of the} \\ \text{true fathers in real triplets).}$$

The likelihood ratio using information  $A$  is then

$$f\left(\frac{Y}{X}\right) = \frac{1-A}{1} = 1-A.$$

By substituting for this in the Essen-Möller formula (1) one  
obtains a "probability of paternity  $W_A$ " (2)

$$W_A = \frac{1}{2-A}.$$

The only information that  $W_A$  contains - besides a neutral  
prior probability - is the exclusion chance  $A$ .

$W_A$  % states how many men among 100 non-excludable men for a  
given mother-child combination are the real fathers;  $100-W_A$  %  
gives the percentage of non-excludable non-fathers.

This assumes that the material on file contains either as  
many fathers as non-fathers or as many cases of kinship as  
of non-kinship.

Because  $A$  can never be negative  $W_A$  is never less than 50%.  
Hence, the information  $A$  can never produce any  $W_A$ -values which  
would speak against paternity - this in contrast to Essen-  
Möller's  $W$ -value ( $W_{EM}$ ).

The information A can be used - to distinguish between the hypotheses X and Y - only in the formulas  $W_A = \frac{1}{2-A}$  (for X) and  $1-W_A$  (for Y). However, by "adapting" A in this way to the Bayes' principle one destroys the "advantage" of greater "clarity" of A. With other words: Once A (the exclusion chance) is transformed to take account of a probability of paternity it becomes "unclear"; for this raises not only the problem of applying statistical behaviour to an individual case but also that of a prior probability. And precisely these are the "difficulties" to understand and to accept Essen-Möller's W-value.

$W_{EM}$  is nothing else but a  $W_A$ -value "corrected" for the individual serotype of the putative father. The father's chance to have contributed to the serotype of the child is largely dependent on the serological similarity between mother and child. For this reason the differences between the values of the two terms are not systematic. In one case  $W_A$  is smaller than  $W_{EM}$  and in another larger - and by varying degrees. The differences can be considerable. E.g. in 14 cases of biostatistically evaluated HLA-A,B findings the value for A was consistently well above 50%, and that for  $W_{EM}$  very much lower (3):

$A\%$	$W_{EM}\%$	$A\%$	$W_{EM}\%$
68	40	89	4.5
80	8.3	91	30
81	9.2	91	41
85	24	95	15
85	31	95	41
88	4.4	95	44
88	42	96	42

In deficiency cases A can be 0;  $W_A$  is then 50% - this in all isolated grandmother and grandfather cases;  $W_{EM}$ , however, can have high values and sometimes also low ones. Apparently, there is no "natural" relation between  $W_A$  and  $W_{EM}$ . However, from a statistical point of view, there is some correlation: as the A-values increase so do the mean  $W_{EM}$ -values. But this is irrelevant in the individual case.

Some supporters of the exclusion chance maintain that at the upper end of the scale A approaches the W-value obtained. In other words, the higher the range the closer the mass of indi-

vidual  $W_{EM}$ -values associated with a specific  $W_A$ -value approaching this value, until they all meet at infinity; and, the lower a  $W_A$ -value the greater the scatter of the associated  $W_{EM}$ -values. Is this true? The prime cause of the scatter of the  $W_{EM}$ -values for fathers and non-fathers - for given child-mother combinations - is the respective homozygosity or heterozygosity of the putative fathers: the  $W_{EM}$ -values of the homozygous men are higher than those of the heterozygous. The scatter is independent of the exclusion chance. Hence one cannot expect that  $W_A$  and the individual  $W_{EM}$ -values will converge at infinity.

$W_{EM}$  contains all the information held in A. The converse, however, is not true; for,  $W_{EM}$  always contains more information than A. For, using  $W_A$  instead of  $W_{EM}$  is tantamount not only to destroying information but also to accepting unsystematic distortions of the reality, i.e. stronger or weaker indications of paternity than are actually the case.

Occasionally a Court questions the evidential value of  $W_{EM}$  when a cohabitor has been named but cannot be found. Such doubts are unwarranted.  $W_{EM}$  applies to the putative father,  $1-W_{EM}$  to the cohabitor, regardless of whether the latter has been named or is only assumed, whether there is only one cohabitor or several, whether the child's mother admits to cohabitation with men other than the putative father or not, or whether proof of cohabitation is provided or not.

Even though  $W_{EM}$  provides full biostatistical information in cases involving a known though missing cohabitor, some experts and judges resort, in addition, to the exclusion chance and argue, e.g., as follows: "If the cohabitor were included in the opinion, the probability of his being excluded from paternity would be e.g. 99%. Hence, one could expect with great certainty that his non-paternity would be established". Conclusions of this nature presume that the cohabitor is in fact not the father - for, as father he could not be excluded. The correct argument in such a situation is the following; "If one regards the defendant with e.g.  $W = 99.73\%$  as the real father of the child, then it can be expected with a probability of e.g.  $A = 99\%$  that the unknown cohabitor will be excluded". This knowledge, however, is useless for the Court's decision.

The answer to the question whether there is any particular advantage in using A (= exclusion chance) or  $W_A$  (= probability of paternity using the exclusion chance as information) in cases of disputed parentage is simple: there is no apparent advantage, neither for normal nor special cases.

#### SUMMARY:

The only correct way to use the exclusion chance A as a sero-statistical parameter is to express it as a probability:

$$W_A = \frac{1}{2-A} .$$

This  $W_A$  however

- is deficient in information in comparison with the full informative Essen-Möller  $W_{EM}$ ;
- is neither easier to understand nor easier to calculate than  $W_{EM}$ ;
- requires - like  $W_{EM}$  - a prior probability;
- cannot interpret  $W_{EM}$ ;
- is unsuitable for setting a decision limit because the  $W_{EM}$ -value of an individual case can be lower than the limit.

$100-W_A\%$  cannot provide a reliable expectation of error in an individual case because the phenotype of the putative father is not taken into full account.

There is no reason that  $W_A$  and  $W_{EM}$  converge in higher degrees.

#### References:

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