Paternity Index (PI)

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As we are discussing the solution to a problem of biostatistical inference it is best to begin by defining the problem. The problem of disputed paternity can be reduced to the following question:

Is this man the father of this child, or not?

Many other questions can be and are addressed during the course of a disputed paternity case, but when the time comes to make a decision this is the only question which remains.

In absolute terms we can never be certain of paternity (or of nonpaternity). It follows that the answer to the above question can only be given in probabalistic terms. The ideal solution to the question posed above is a conditional probability of the form

P(paternity/evidence), the posterior probability of paternity.

During the course of the evaluation of disputed paternity a great many statistics can be defined, and a great many probabilities related to these statistics can be computed, but only the posterior probability of paternity (and its complement) are of direct relevance to the question before the court. In general, it is uncommon for a problem in biostatistical inference to admit to a solution in the form of a posterior probability. When such a solution is not possible, a variety of indirect biostatistical methods can be applied, none of which directly address the question. Whatever the biostatistical method utilized it is customary to define a null hypothesis (H_0) and counterhypothesis (H_1). For the problem of disputed paternity, these are:

Advances in Forensic Haemogenetics 1
Edited by B. Brinkmann and K. Henningsen

H_0: The man is a nonfather of the child

H_1: The man is the father of the child.

The paternity index is defined as

$$PI = \frac{P(\text{phenotypes of trio}/H_1)}{P(\text{phenotypes of trio}/H_0)}$$

The PI is a classical likelihood ratio which has a large number of interesting properties, of which only three will be discussed here. The first two are:

1. In the typical case, both hypothesis are counterhypothesis are simple, in the sense that each completely specifies the frequency distribution of the phenotypes of the trio. Given the phenotypes of the trio the PI is, therefore, a number free of any arbitrary parameters.
2. Given the prior probability of paternity and the PI application of the rules of conditional probability yields W, a conditional probability of the form

   W = P(paternity/prior, PI)

   Comparison of this expression with the ideal solution to the problem of disputed paternity indicates that W is, in fact, that solution. Moreover, as the prior probability is independent of the phenotypes of the trio, the evidence relating to the question before the court has been factored: the PI contains all of the relevant <u>genetic</u> information relating to the probability of paternity.

All of the above has been known for many years. It is natural to ask: why is there still controversy? Other than confusion caused by failure to comprehend the fundamental aspects of the problem, there appears to me to be three contributing factors:

1. Most statisticians are accustomed to inferential problems about which only incomplete and indirect solutions can be obtained. Some of them make the

expert's error of assuming that the problem of disputed paternity is subject to similar limitations, when, in fact, the ideal solution can be obtained in a straightforward manner. Thus, they bring their biostatistical cannons to attack the impregnable wall of the fortress, when in truth they possess the biostatistical key which unlocks the front gate.

2. Other statistics which can be computed in disputed paternity cases are intuitively attractive. For example, the observation that an alleged father is not excluded by a powerful combination of genetic tests is powerful intuitive evidence that he is, in fact, the biological father. It is thus natural to conclude that the exclusion probability (A) is an important statistic in disputed paternity. On the other hand, if the man is nonexcluded, he is either the father or a non-excluded nonfather. This issue also needs to be addressed. One way to do so is to examine the distribution of the PI for fathers, or for nonfathers, or for non-excluded nonfathers. This leads naturally to computation of inverse ("tailed") probabilities and to inferences of the Neyman-Pearson type.

3. Many statisticians don't like prior probabilities. They correctly view such a probability as a probability concerning an event. As in an absolute sense every event is unique, one can assert that the concept of probability does not apply to events. This seems to me to be an excessively academic point of view. In our daily lives we are accustomed to making inferences regarding events; events which may be regarded as unique. We are also accustomed to expressing and understanding our expectations of unique events in terms of probabilities. Thus, when the television meteorologist announces that the chance of rain this

weekend is 30%, the switchboard does not light up at the television station with complaints by statisticians that the concept of probability is inapplicable to this unique event, and nobody calls the station to complain that they do not understand the weather report. Dislike of priors explains in part the attractiveness of tailed probabilities, which are prior free. However, one can ignore priors only at the cost of ignoring the issue at hand as the following examples will demonstrate.

Suppose a coin is presented. The question to be addressed is whether or not the coin is biased; the evidence to be evaluated consists of a trial of flipping the coin. The null hypothesis (H_0) is that the coin is true (i.e. when flipped, the probability of heads and probability of tails is each ½). If the experimental result is that the coin produced ten heads in ten flips, the null hypothesis can be tested by classical Neyman-Pearson methods as follows:

$$P(\geq 10 \text{ heads in } 10 \text{ flips/true coin}) = 2^{-10} \backsim 10^{-3}$$

This result can be labeled "significant" or whatever, and the null hypothesis can be "rejected" or not, but a meaningful statement regarding the truth of the null hypothesis is not possible, in part because the counterhypothesis has not been tested. In this case, the counterhypothesis H_1(the coin is biased) cannot be tested because it is not a simple hypothesis. To say that the coin is biased is not to specify in what manner it is biased. For example, if "biased" means "biased in favor of tails" then the experimental result (10 heads in 10 flips) actually favors the null hypothesis that the coins is true, despite the "significance" of the result.

We can create a simple counterhypothesis H_1: "the coin has two heads." Now a likelihood ratio can be computed as follows:

$$L = \frac{P\,(\,10 \text{ heads in } 10 \text{ flips}/H_1)}{P\,(\,10 \text{ heads in } 10 \text{ flips}/H_0)} = 2^{-10} \backsim 10^{-3}$$

A meaningful statement regarding the truth of the null hypothesis is still not possible. If, for example, Professor Nijenhuis were to hand me a coin, inform me that it was either true or two-headed, and I were to obtain 10 heads in 10 flips, I would conclude that the coin probably is two-headed because knowing Professor Nijenhuis, under such circumstances I would judge the prior probability of a two-headed coin to be substantial. On the other hand, were the coin to come from my pocket and I were to obtain the same result, I would conclude that the coin is probably <u>not</u> two-headed because over the years I have pulled many thousands of coins from my pocket and have yet to encounter one with two heads.

As further example, consider a null hypothesis tested by χ^2(a typical tailed probability method). If the tailed probability is less than 0.05 without further assumptions one can "reject" the null hypothesis, as this is not a conclusion about the truth of the null hypothesis, but only a statement about the tail value. However, one cannot logically conclude that the null hypothesis is unlikely to be true without assuming that the prior probability of the null hypothesis is significantly less than unity.

These examples demonstrate that meaningful statements regarding the truth of hypotheses cannot be made without explicit or implicit statements regarding prior probabilities.

As the court has the responsibility in a disputed paternity matter to make meaningful statements regarding the hypotheses, the court must consider the prior probability of paternity. Thus, there is no justification for withholding the paternity index from the court, as the posterior resulting probability of paternity directly addresses the issue before the court.

The question then arises: Should additional statistics be submitted to the court? Can the court improve on its decision making process if it has, in addition to the PI, the exclusion probability (A) and/or information regarding the issue of father vs non-excluded nonfather (e.g. tailed probabilities)? These questions can be answered by considering the following example. Suppose we are given two cases:

Case #1	Case #2
PI = 99	PI = 99
A = 0.999	A = 0.90

and the question is posed: In which case is the biostatistical evidence in favor of paternity stronger? The exclusion probabalist would argue as follows: In terms of PI, there is no difference. The alleged father in Case #1, however, was subjected to a more powerful exclusionary battery (a statement with which we all should agree). The exclusion probabalist would conclude that the evidence in Case #1 is stronger than in Case #2.

The tailed probabalist would argue as follows: In terms of PI, there is no difference. The alleged father in Case #1, however, looks more like a non-excluded nonfather because the tail of the distribution of the PI for non-excluded nonfathers is larger for Case #1 than for Case #2 (a statement with which we all should agree). The tailed probabalist would conclude that the evidence in Case #2 is stronger than that in Case #1.

It is clear that the tailed probabalist and the exclusion probabalist cannot both be right. There is no point in trying to decide which argument is correct, as they have equal merit. This can be appreciated by considering the third important property of the PI; it can be factored:

$$PI = \frac{1}{1-A} \cdot \frac{P(\text{phenotypes of trio/paternity,nonexclusion})}{P(\text{phenotypes of trio/nonpaternity,nonexclusion})}$$

The first term contains only the exclusion probability (A). The second term, which looks very much like the PI, is the likelihood ratio which tests the hypothesis that a nonexcluded man is the father relative to the hypothesis that he is a nonfather. As it is computed only with regard to trios in which the alleged father is not excluded, the second term is independent of the exclusion probability (A). The exclusion probabalist is arguing that the first term is larger for Case #1 than for Case #2; the tailed probabalist is arguing that the second term is larger for Case #2 than for Case #1. These two terms are like the sides of a rectangle.

Suppose two rectangles are presented. The first measures 6 units by 4 units; the second measures 8 units by 3 units. The question is posed: Which rectangle is larger? The straightforward approach is to note that the two rectangles are equal in area, so the question is moot. One can imagine that the following argument could be made by a "length probabalist": "On the basis of area there is no difference. However, length is important in determining the size of a rectangle, so that rectangle #2 must be larger." The reader no doubt can produce the argument that would be made by a "width probabalist."

Of course, these arguments are silly. Once the area has been computed, the length and width need not be considered again. In the same way, while the exclusion probability (A) may be important on the evaluation of disputed paternity, it is incorporated into the PI. Once the PI has been computed, it would be unnecessary and misleading to consider A again. The same is true of the issue of father versus non-excluded nonfather.

For those who come to the field of disputed paternity from an experimental, rather than mathematical, background I offer the following observations. The paternity index, being a quotient of population frequencies, can be obtained directly from experimental data, as well as by computation from gene and haptotype frequency tables. The posterior probability of paternity is a mathematical expectation. This expectation can be experimentally verified for actual case material by testing a number of cases with a limited test battery, noting the expectation of paternity, and confirming that expectation by testing in further genetic systems. Confirmation can also be obtained by computer simulation (Monte Carlo methods). Despite several decades of controversy regarding paternity probability, no practical advantage has ever been shown for any biostatistical summary of genetic evidence different from, or in addition to, the paternity index. We should heed the old adage "If it's not broken, don't fix it."